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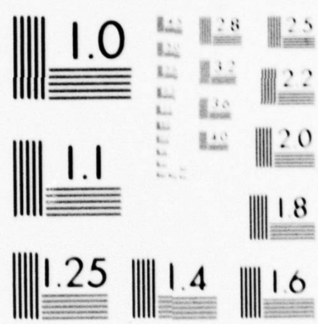

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by

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A.G. Purcell

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Procurement Executive, Ministry of Defence
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17. Abstract

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SUMMARY

The response of an aircraft as a function of time on encountering an isolated ramp gust is derived from its response to a unit step gust. Two FORTRAN programs are described which treat separately straight ramp and smooth ramp (one-minus-cosine) gust profiles. The (critical) gust length causing maximum dynamic response is determined and responses to simple gust patterns (pairs) are investigated.

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LIST OF CONTENTS

1 INTRODUCTION

The problem discussed in this Report is the following:

Given the response F(t) of an aircraft flying at a certain speed to a unit step gust, predict the response to a family of ramp gusts and determine the critical gust length at which the extreme response occurs.

The investigation was treated, purely as a mathematical and computational exercise; the soundness and applicability of the physical theory (*eg* Jones[1]) was taken for granted. Jones recalls in that Report that the earliest airworthiness requirements were based on response to discrete gusts but that recently more emphasis has been laid on irregular turbulence, the implication being that responses can be deduced adequately from stationary random process theory. He then argues that large aircraft loads cannot be satisfactorily predicted by spectral analysis and that the consideration of discrete ramp gusts is preferable if extreme responses are to be reliably estimated. Effectively, the argument is that the power spectrum (PSD) method estimates the root mean square response whereas one is really interested in the extreme behaviour and that the non-Gaussian character of turbulence can be so marked as to undermine the usefulness of the rms value. In particular, the PSD method is not to be trusted when the dominant aircraft mode is well damped.

The problem is the familiar one that the weight of interest falls in the tail of a distribution where probability levels are only poorly defined by empirical evidence. Any extrapolation from the relatively well defined body of the distribution is unsafe unless independent information on the shape of the distribution is available. In addition, Jones points out that dynamic response is produced by *changes* in wind velocity so that one must determine the probability distribution of velocity *gradients*. Many power spectrum investigations have only considered the velocity distribution.

To cope with situations in which non-Gaussian velocity gradients are significant, Jones has developed a statistical discrete gust model wherein the intensity, w_H of a ramp gust is related to its gradient distance (H) by

$$w_H = w_0 H^{\frac{1}{3}} \qquad (1)$$

where w_0 is derived directly from measurements and varies with altitude. This dependence of gust intensity on gradient length is valid over the range of wave-
165 lengths (λ) for which the Kolmogoroff (proportional to $\lambda^{\frac{5}{3}}$; $\lambda \approx 2H$) power spectrum of atmospheric turbulence applies. Fortunately, the Kolmogoroff spectrum

appears to be valid far beyond the wavelengths associated with extreme aircraft response.

The ramp (see Fig 1) may be either straight or 'one-minus-cosine' (smooth).

$$w_1(x) = w_H\left(\frac{x}{H}\right) \qquad 0 \leqslant x \leqslant H \qquad (2)$$

$$w_2(x) = \tfrac{1}{2}w_H\left(1 - \cos\frac{\pi x}{H}\right) \; ; \qquad 0 \leqslant x \leqslant H \; . \qquad (3)$$

The latter has the advantage that $\frac{dw_2}{dx} = 0$ at $x = 0, H$ (whereas $\frac{dw_1}{dx}$ is discontinuous at the end points) and is generally preferred.

In the next section we derive the ramp response from the unit step response and the gust profile. Later sections explain the computational details and the determination of that gust length, $\bar{H}$ which causes the worst response, $\bar{\gamma}$. This is extended in section 7 to deduce the most adverse gust pair.

Although the problem appears straightforward at first sight, being basically one of numerical integration and interpolation there are complicating features. In addition, although the physical model is only approximate, we did not wish to further degrade it with rough calculations. Our efforts to ensure efficiency, accuracy and stability in the numerical operations are duly described.

Thus, in section 3 we describe the integration procedure based on cubic spline fitting of the step response, explaining the considerable economies which are possible with this formulation. The straight ramp generates a discontinuity in the slope of the ramp response which can be most troublesome (if, for example, the 'corner' is smoothed) if not properly handled as described in section 4. Extra trial gust lengths are needed to refine the estimate of the critical gust, $\bar{H}$, and more especially, the critical time, $\bar{t}$. These are kept to a minimum by an efficient interpolation procedure (see section 5).

The end result, we believe, are computer programs which can determine the critical ramp gust and gust pair with errors which are negligible compared to those arising from the fundamental physical assumptions and idealisations.

2 DERIVING THE RAMP RESPONSE FROM THE STEP RESPONSE

2.1 Assumptions 165

The important assertion which motivated the present work is that the response to an isolated ramp gust of a given length and profile shape is a meaningful quantity to calculate. Jones[1] claims that such a representation is usually

reasonable at least in the vicinity of the critical gust length (*ie* that isolated gust which generates the worst behaviour). In other words, despite turbulence being comprised of gusts of all lengths, only a limited range is important and the magnitude of the worst response can be fairly well predicted by identifying the critical isolated gust (or, more generally, sequence of gusts).

A second major assumption is that the system is linear. A corollary is then that

$$\gamma(w_0, H) = w_0 \gamma(1, H)$$

where γ is the (extreme) response.

The linearity assumption also facilitates consideration of gust pairs whereby overswing tuning can result in a greater response than any single gust would cause. The effect, of course, is unimportant for well damped modes. Gust pairing is therefore an extension of the basic concept to moderately damped modes and involves finding the critical gust length, $\bar{H}_0$, which results in the worst overswing as well as that ($\bar{H}$) which gives the extreme primary response. The most adverse situation is when the two gusts occur with opposite sign and appropriate spacing H_s (see Fig 2 and section 7) so that the first overswing due to the first gust coincides with the primary peak due to the second gust. The spatial extent of the critical gust pattern is therefore

$$\bar{H}_2 = \bar{H} + \bar{H}_0 + H_s \ . \tag{4}$$

More complex sequences of gusts are not considered. However, it is possible to extend the concept to include very lightly damped modes and a recent paper by Jones[2] envisages the superposition of responses to as many as eight successive gusts. Of course, the probability of the critical pattern occurring decreases as the number of component gusts is increased.

2.2 Theoretical considerations

Let the step response be $F(t)$ and the ramp response be $\phi(H,t)$. Essentially, the calculation of ϕ involves the numerical integration of $F(t)$. Note that strictly speaking we should write $F(v,t)$ but it is customary to regard the aircraft velocity as constant during an encounter with turbulence. It is not a simple (*ie* linear) matter to determine ϕ (and hence $\gamma(H)$) at
165 different speeds - $F(t)$ must be recalculated.

A ramp gust can be thought of as a series of n step gusts (see Fig 3). Thus we have to combine together up to n step responses, each scaled and appropriately time shifted. The linearity assumption is essential if the means of combination is to be simple addition. The response at time t' will be given by

$$\phi(H,t') = \sum_{i=1}^{j} \delta w_i F(t' - \tau_i) \tag{5}$$

where $n\delta t = H/v$; $\tau_i = i\delta t$; $j = \min(n, t'/\delta t)$. Letting $\delta t \to 0$,

$$\phi(H,t') = \int_0^{\min(t',H/v)} \left.\frac{dw}{dt}\right|_\tau F(t' - \tau)d\tau \ . \tag{6}$$

Or, making the transformation $t = t' - \tau$,

$$\phi(H,t') = \int_{\max(0,t'-H/v)}^{t'} \left.\frac{dw}{dt}\right|_{t'-t} F(t)dt \ . \tag{7}$$

The limits reflect the assumption that $\frac{dw}{dt} = 0$ when $t < 0$ or $t > H/v$. Expressed as functions of time and setting $w_0 = 1$, equations (2) and (3) become

$$w_1(t) = \frac{vt}{H^{2/3}} \ ; \ w_2(t) = \tfrac{1}{2}H^{1/3}\left(1 - \cos\frac{\pi vt}{H}\right). \tag{8}$$

Hence (7) gives, respectively,

$$\boxed{\begin{aligned} \phi_1(H,t') &= \frac{v}{H^{2/3}} \int_{\max(0,t'-H/v)}^{t'} F(t)dt && (9) \\ \phi_2(H,t') &= \frac{\pi}{2}\frac{v}{H^{2/3}} \int_{\max(0,t'-H/v)}^{t'} \sin\frac{\pi v}{H}(t' - t)F(t)dt && (10) \end{aligned}}$$

Thus the straight ramp response at time t' is merely the area under the step response between t' and some earlier time, either zero or $t' - H/v$. Furthermore, differentiating (9) gives

$$\frac{\partial \phi_1}{\partial t'} = \frac{v}{H^{4/3}}\left(F(t') - \begin{Bmatrix} 0 \\ F(t' - H/v) \end{Bmatrix}\right) \tag{11}$$

$$\frac{\partial \phi_1}{\partial H} = -\frac{2}{3H}\,\phi_1(H,t') + \begin{Bmatrix} 0 \\ \dfrac{F(t' - H/v)}{H^{4/3}} \end{Bmatrix}. \tag{12}$$

From equation (11) we can envisage three possible situations concerning the time, t_γ, at which the maximum response, $\gamma(H)$ occurs. As depicted in Fig 4 t_γ can be greater than, less than, or equal to H/v. The first two present no computational difficulties because $\partial\phi_1/\partial t = 0$ but the third case could be troublesome because the discontinuity in $\partial\phi_1/\partial t'$ coincides with t_γ.

Now, in any numerical scheme integrals can only be computed at discrete (usually pre-determined) points. If one seeks maxima and minima of a function defined only as a set of points then some interpolation is necessary. Of necessity the interval between adjacent points will be represented by a fully continuous interpolating function and difficulties can be expected if the original function is less well behaved.

Trouble was indeed encountered with the straight ramp when the derivative discontinuity at $t = H/v$ fell within the same integration interval as t_γ. The resulting interpolation errors superimposed a ripple on the $\gamma(H)$ curve making impossible the accurate determination of $\bar{H}$. The problem only arises if $F(0) \neq 0$ but unfortunately this is common because $F(t)$ is the response to a *step* gust. The counter-measures adopted are described in section 4. There was no corresponding difficulty with the one-minus-cosine ramp because $\partial\phi_2/\partial t$ is everywhere smooth.

It can also be surmised that t_γ will increase with H until the first zero, $\tilde{t}_0$, of F is reached. For $H > v\tilde{t}_0$ one would expect $t_\gamma = \tilde{t}_0$ (*ie* case 2 of Fig 4) and $\gamma(H)$ to decrease as $H^{-1/3}$. Equation (12) provides the condition to be satisfied at $\bar{H}$. First we observe that $\bar{t} < \bar{H}/v$ is impossible.
165 For $\bar{t} > \bar{H}/v$ we have

$$\bar{\gamma} = \frac{3}{2}\bar{H}^{-1/3}F(\bar{t} - \bar{H}/v).$$

But from (11),

$$F(\bar{t}) = F(\bar{t} - \bar{H}/v) \ . \tag{13}$$

Thus

$$\bar{\gamma} = \frac{3}{2}\bar{H}^{\frac{1}{3}}F(\bar{t}) \ . \tag{14}$$

If $\bar{t} = \bar{H}/v$ then neither derivative (11) or (12) is separately zero but we still have

$$\frac{\partial\phi_1}{\partial t'} + v\,\frac{\partial\phi_1}{\partial H} = 0$$

giving $\bar{\gamma} = \frac{3}{2}\bar{H}^{\frac{1}{3}}F(\bar{H}/v)$.

Hence equation (14) remains valid when $\bar{t} = \bar{H}/v$. It is useful for checking purposes since the computer program does not use it directly but instead locates $\bar{H}$ by interpolation.

Equation (9) for the straight ramp gust encounter can be solved very economically because the integrand is independent of H . One merely has to store once and for all the accumulated area under $F(t)$ at predetermined time points and then use appropriate bandwidths when considering different H . Equation (10) is far more complicated because the step response is weighted by an H-dependent factor. Furthermore, practical checking formulae analagous to (13) and (14) cannot be given.

3 EVALUATION OF INTEGRALS

The first operation in both the straight ramp and the smooth ramp programs is to fit a cubic spline to $F(t)$. The two spare degrees of freedom are taken up by third derivative continuity at the second and penultimate points (knots). Subroutine TB04A of the Harwell Subroutine Library is employed for this purpose. A particular virtue of the cubic spline is that the area between knots is easily calculated and will be accurate to third order even when the abscissae are unequally spaced (*cf* Simpson's rule).

$$A_i = \frac{\Delta t}{2}(F_{i+1} + F_i) + \frac{\Delta t^2}{12}(F'_i - F'_{i+1}) \tag{15}$$

where $F' = dF/dt$. The integral in (9) is therefore readily evaluated while that in (10) requires further manipulation as follows.

Using the cubic spline, the 'weighted' (in the sense of (10)) area between two adjacent knots is given by

$$\int_{t_0}^{t_1} \sin \alpha(t' - t)F(t)dt = \left[\frac{F(t)}{\alpha} \cos \alpha(t' - t) + \frac{F'(t)}{\alpha^2} \sin \alpha(t' - t) - \frac{F''(t)}{\alpha^3} \cos \alpha(t' - t) - \frac{F'''(t)}{\alpha^4} \sin \alpha(t' - t)\right]_{t_0}^{t_1} . \quad (16)$$

If we now consider the total integral over m intervals, the fact that F, F' and F'' are continuous at the knots and that F''' is constant on each interval causes most terms to cancel leaving

$$\int_{t_0}^{t_m} \sin \alpha(t' - t)F(t)dt = \left[\frac{F(t)}{\alpha} \cos \alpha\,(t' - t) \ldots\right]_{t_0}^{t_m} + \frac{1}{\alpha^4} \sum_{i=1}^{m} F_i''' (\sin \alpha\,(t' - t_{i-1}) - \sin \alpha(t' - t)) . \quad (17)$$

Now, from equation (10) we have $\alpha = \pi v/H$, $t_m = t'$. Hence $\sin \alpha(t' - t_m) \equiv 0$, $\cos \alpha(t' - t_m) \equiv 1$ while at the lower integration limit we must distinguish the two cases (i) $t_0 = t' - H/v > 0$; (ii) $t_0 = 0$.

In the first case we have:

(i) $\sin \alpha(t' - t_0) = 0$, $\cos \alpha(t' - t_0) = -1$.

Therefore the first term on the right hand side of (17) becomes

$$\frac{F(t') + F(t' - H/v)}{\pi v/H} - \frac{F''(t') + F''(t' - H/v)}{(\pi v/H)^3} . \quad (18)$$

Regarding the second term on the right of (17), all intervals except possibly the first (depending whether or not H/v is an integral multiple of Δt) will be of the same size, Δt . Therefore,

$$\sum_{i=1}^{m} = \sum_{i=2}^{m} F_i''' (\sin \alpha(t' - t_{i-1}) - \sin \alpha(t' - t_i)) + F_1''' (\sin \alpha(t' - t_0)$$

$$- \sin \alpha(t' - t_1))$$

where F_i''' is the third derivative of $F(t)$ in the ith interval following t_0. But $t_0 = t' - H/v$, $t_1 - t_0 = \Delta t_1$ (say), $t' - t_i = (m - i)\Delta t$ and

$m = \frac{H}{v\Delta t}$ rounded up if necessary.

Therefore $$\sum_{i=1}^{m} = \sum_{i=2}^{m}\left(F_i''' \, 2 \cos \alpha(t' - t_i + \frac{\Delta t}{2}) \sin \frac{\alpha\Delta t}{2}\right) - F_1''' \sin \alpha\Delta t_1$$

$$= 2 \sin \frac{\pi v\Delta t}{2H} \sum_{i=2}^{m}\left(F_i''' \cos \frac{\pi v}{H} (m - i + \tfrac{1}{2})\Delta t\right) - F_1''' \sin \frac{\pi v\Delta t_1}{H} .$$

$$\qquad \ldots\ldots \quad (19)$$

In the second case we have:

(ii) All intervals are of length Δt and the integrand does not vanish at the lower limit. The first term on the right of (17) becomes

$$\frac{F(t')}{\alpha} - \frac{F''(t')}{\alpha^3} - \frac{F(0)}{\alpha} \cos \alpha t' + \frac{F''(0)}{\alpha^3} \cos \alpha t' - \frac{F'(0)}{\alpha^2} \sin \alpha t' \qquad (20)$$

while, following the above analysis, the sum reduces to

$$2 \sin \frac{\pi v\Delta t}{2H} \sum_{i=1}^{m} F_i''' \cos \frac{\pi v}{H} (m - i + \tfrac{1}{2})\Delta t \qquad (21)$$

where $m = t'/\Delta t$.

It is therefore necessary to evaluate F'' at each knot and F''' on each interval and to store them. In addition, in order to evaluate efficiently the integrals for all t' at given H it is helpful to first store

165

$$\cos (j - \tfrac{1}{2}) \frac{\pi v\Delta t}{H} \quad \text{for } j = 1,2 \ldots \text{ integer part of } \left(\frac{H}{v\Delta t}\right).$$

The only quantities which then have to be recalculated at each time point are *cos* $\alpha t'$ *and* *sin* $\alpha t'$ (for use in (20)). When $t' > H/v$ even this becomes unnecessary although $F(t' - H/v)$ and $F''(t' - H/v)$ may then have to be specially interpolated from the cubic approximation over the relevant interval for use in (18).

Potential trouble spots in the above formulation are the heavy dependence on third derivatives and the presence of $F'(0)$ and $F''(0)$ in (20) because no restrictions are placed on the derivatives generated at the end points of the spline. Tests with analytic step responses have verified that the numerical scheme can be very accurate but ultimately of course, in any practical case much will depend on the nature of $F(t)$ and the sampling interval, Δt (see also section 8). Perhaps the more important feature of the comparison with analytic expressions was the validation of the programming of equations (18) to (21).

4 OBTAINING $\gamma(H)$

The previous two sections explained how the ramp response is calculated at equally spaced time points for each trial gust length, H. We now have to detect its primary peak and, where applicable, the maximum overswing. No assumptions can be made about the shape of the curve which makes the location of global extrema all the more difficult. The basic procedure in both programs is to search the set of $\phi(t)$ for the largest positive and negative values and to then interpolate $\gamma(H)$ and t_γ using $d\phi/dt$. The situation depicted in Fig 5 will therefore be wrongly analysed as shown but it is hoped that Δt can be specified sufficiently small to render such a shortcoming unimportant.

As noted in section 2, when using the straight ramp model we have to be particularly careful to evaluate $\phi(H/v)$ explicitly because $d\phi/dt$ is there discontinuous. At other absicissae $d\phi/dt$ can be found directly from the tabulated step response using (11). We therefore have to consider the following possibilities.

(1) $\phi(H/v)$ is the largest ordinate found initially,

then either (a) $d\phi/dt$ changes sign at H/v in which case $t_\gamma = H/v$, or

(b) $d\phi/dt$ does not change sign at H/v, *ie* t_γ lies between H/v and one or other of the adjacent abscissae.

(2) $\phi(H/v)$ is not the largest ordinate found initially but $j\Delta t$ is. Then t_γ lies somewhere between $(j - 1)\Delta t$ and $(j + 1)\Delta t$. Again, if H/v lies within this interval, care must be taken to locate the nearest point to $j\Delta t$ which

brackets the peak so that t_γ and $\gamma(H)$ can be accurately interpolated, *ie* t_γ lies in one of the intervals $((j-1)\Delta t, j\Delta t)$, $(j\Delta t, (j+1)\Delta t)$, $(H/v, j\Delta t)$, $(j\Delta t, H/v)$.

The process of interpolating the peak is merely that of finding a stationary point of a cubic given the function value and first derivative at two bracketing abscissae.

The smooth ramp model differs in that $t = H/v$ does not present any special difficulties but on the other hand, obtaining $d\phi/dt$ is not so straightforward. In fact differentiating (10) gives

$$\frac{\partial \phi_2}{\partial t'} = \frac{\pi^2 v^2}{2H^3} \int_{\max(0,t'-H/v)}^{t'} \cos \frac{\pi v}{H} (t' - t) F(t) dt \ . \tag{22}$$

Instead of evaluating the above integral we have elected to fit a cubic spline through $\phi_2(t)$ and to interpolate the required extrema from it having located the maximum and minimum tabulated points by inspection as in the straight ramp model.

5 FINDING $\bar{H}$, $\gamma(\bar{H})$ AND $t_\gamma(\bar{H})$

Both the primary peak of $\phi(t)$ and the largest overswing are determined for each trial value of H supplied by the user. Actually, we seek the extreme positive and negative responses $\gamma_+(H)$, $\gamma_-(H)$. It was originally suggested that we call the larger (in absolute value), γ_+ and the smaller, γ_- but this seemed to be more confusing and could (if $\gamma_{+(H)}$ and $|\gamma_-(H)|$, as we have defined them, intersect) complicate the shape of $\gamma(H)$ causing discontinuities in $d\gamma/dH$ and corresponding difficulty in locating $\bar{H}$ (see Fig 6). Using the revised convention, $\gamma(H)$ is more likely to be unimodal and only after $\bar{\gamma}_+$ and $\bar{\gamma}_-$ have been estimated do we decide which is the primary peak and which is the overswing. It is not necessarily the case that the sign of the primary is the same as that of the first peak in the ramp response nor that the worst overswing occurs after the primary.

In order to find $\bar{H}$ it is essential that the user supplies trial values of 165
H which bracket $\bar{H}$, or more precisely, that the extreme $\gamma(H_{trial})$ occurs neither at the smallest nor the largest gust length supplied. In principle the

program could select its own trial values of H, choosing gust lengths anywhere between $v\Delta t$ and vt_{max} (where t_{max} is the last point of the given step response). In practice it was felt that the response to specific gusts could be of interest so the programs expect a minimum of three trial gust lengths. The user should avoid $H < v\Delta t$ or $H > vt_{max}$ and should be careful to supply enough time history, $F(t)$. Strictly speaking, sufficient step response should be input to enable $\gamma(2\bar{H})$ to be calculated but this may not always be possible (see section 6).

Assuming these conditions have been fulfilled we will get a best estimate of $\bar{H}$ and lower and upper bounds H_L and H_U. We then use a safeguarded quadratic interpolation scheme outlined below to successively improve $\bar{H}$ until $max(H_U - \bar{H}, \bar{H} - H_L)$ is sufficiently small. Several further ramp responses will have to be calculated but we feel that the labour is justified. It may be thought possible to economise calculation by only evaluating ϕ in the vicinity of t_γ but unfortunately t_γ can change discontinuously (and hence $\bar{t}$ cannot safely be interpolated) as the following example shows.

Consider the step response shown in Fig 7a in conjunction with the straight ramp model and let area A3 exceed area A1 and $t_{03} - t_{02} > t_{01}$. Then (see Fig 7b) $t_\gamma = H/v$ until $H = vt_{01}$ is reached after which $t_\gamma = t_{01}$ until H is sufficiently large that A3b = A1 when it switches discontinuously to t_{13} increasing to t_{03} when $H = v(t_{03} - t_{02})$. Eventually t_γ will revert to t_{01} at t_{14}

where $$\int_{t_{14} - H/v}^{t_{14}} F(t)dt = A1 .$$

This is a powerful argument in favour of computing extra ramp responses rather than attempting to interpolate $\bar{t}$ directly from the responses at the given values of H. Although $\bar{\gamma}$ and $\bar{H}$ may perhaps be satisfactorily derived from the initial set it is apparent that $\bar{t}$ could be grossly in error and hence the most adverse gust pair predicted (see section 7) would be totally misleading.

We now describe the quadratic approximation procedure used to improve $\bar{H}$. The simplest algorithm would be one which always maintained three points bracketing the peak so that interpolation was always possible. However, if the curve was very asymmetric such a process would be inefficient. For example, in Fig 8,
165 point 3 would always be retained and slow convergence would be experienced because

of its remoteness from $\bar{H}$. A preferable scheme is to maintain wherever possible the three highest points even if it means extrapolating $\bar{H}$. An artificial bound would then be needed to safeguard the extrapolation. Such algorithms are commonplace in one-dimensional search routines appearing in optimisation programs and are discussed by Brent[3] and Gill and Murray[4]. If $\tilde{H}$ is the highest point so far found and H_L and H_U are the current bounds on $\bar{H}$ then the artificial bound is defined by

$$\left.\begin{aligned} H_A &= \tilde{H} + \beta(H_L - \tilde{H}) && \tilde{H} \geqslant \mu \\ &= \tilde{H} + \beta(H_U - \tilde{H}) && \tilde{H} < \mu \end{aligned}\right\} \tag{23}$$

where $\mu = \frac{1}{2}(H_L + H_U)$; $\beta = \frac{1}{2}(3 - \sqrt{5})$.

If the extrapolation predicts a point outside (H_L, H_U) then H_A is used instead. This strategy becomes comparable with the usual bracketing technique as the skewness of the curve decreases. In fact, the programs work with the curve $\gamma(\log H)$, making the determination of $\bar{H}$ even easier. $\bar{t}$ is not interpolated but is obtained directly from $\phi(\bar{H},t)$ as of course is $\bar{\gamma}$.

A further refinement prevents the evaluation of ϕ at values of H which are too close together. The user supplies a parameter specifying the relative accuracy, $\Delta H/\bar{H}$, required and the program locates $\bar{H}$ such that

$$\max(\log_e(\bar{H}/H_L),\ \log_e(H_U/\bar{H})) < \Delta H/\bar{H} \tag{24}$$

while keeping the minimum separation between any pair of (logarithmic) trial gust lengths $\geqslant \frac{\Delta H}{2\bar{H}}$.

6 SENSITIVITY FACTOR

Jones[1] defines λ , the gust length sensitivity, a measure of the sharpness of the peak in $\gamma(H)$ by

$$\lambda = \frac{1}{\log_e 2}\left(\frac{2\gamma(\bar{H}) - \gamma(2\bar{H}) - \gamma(\bar{H}/2)}{2\pi\gamma(\bar{H})}\right)^{\frac{1}{2}} . \tag{25}$$

An alternative expression which ignores gusts longer than $\bar{H}$ is

$$\lambda_W = \frac{1}{\log_e 2}\left(\frac{\gamma(\bar{H}) - \gamma(\bar{H}/2)}{\pi\gamma(\bar{H})}\right)^{\frac{1}{2}} . \tag{26}$$

These quantities are evaluated for both $\gamma_+(H)$ and $\gamma_-(H)$.

Note that the calculation of $\gamma(2\bar{H})$ may require the provision of an uneconomic amount of step response. Likewise, the user should beware of suggesting too short a trial gust length or of occasions where $\bar{H}$ is so short that $\bar{H} < 2v\Delta t$. The programs will ignore trial gust lengths $< v\Delta t$ but will attempt to determine $\gamma(\bar{H}/2)$ in similar circumstances so that λ may be evaluated. Note that for ramp gusts of very short gradient distance (straight or smooth) we have the asymptotic result

$$\phi(H,t) \sim H^{\frac{1}{3}}F(t) \qquad \text{as } H \to 0 . \tag{27}$$

Thus a unit step gust can effectively be regarded as being equivalent to a ramp gust of length 1 ft. However, the programs do not exploit this feature, *ie* the maximum and minimum of $F(t)$ are not determined.

7 GUST PAIRS

Having calculated $\bar{\gamma}_+$, $\bar{t}_+$, $\bar{H}_+$, $\bar{\gamma}_-$, $\bar{t}_-$ and $\bar{H}_-$, we identify the primary peak with the larger of $\bar{\gamma}_+$ and $|\bar{\gamma}_-|$, the smaller becoming the worst overswing. We then deduce the most adverse gust pair situation as follows.

The maximum possible response is $(\bar{\gamma}_+ + |\bar{\gamma}_-|)$ occurring at $\bar{t}_+$ or $\bar{t}_-$ whichever is the later. Let us assume for clarity that $\bar{t}_- > \bar{t}_+$. Then, as noted in section 2, the worst gust pair is a ramp gust of length $\bar{H}_-$ followed by one of opposite sign of length $\bar{H}_+$, separated by H_s where (see Fig 2)

$$H_s = v(\bar{t}_- - \bar{t}_+) - \bar{H}_- . \tag{28}$$

It is conceivable that $H_s < 0$ will result but again we leave aside the interpretation of such a situation.

8 ACCURACY

As stated at the beginning of this paper, we have tackled a specific mathematical problem leaving it to others to assess the results and the validity of the models used. The interaction with our computer programs really only occurs through the specification of $\Delta H/\bar{H}$. The fixing of this parameter depends primarily on one's assessment of the accuracy of the discrete ramp model. Having made such an appraisal it is then up to the user to ensure that the step response is sufficiently well represented for the desired accuracy to be attained for it must be remembered that the numerical scheme has a finite accuracy and spline
165 fitting, integrating and interpolating are all sources of error. The question then is - how close is the problem which the program has accurately solved to

that which one has tried to specify? In a practical situation the only insight available is by rerunning the program with fewer points defining the step response and comparing results. If they are too different then a finer tabulation of $F(t)$ is probably necessary.

Both the straight ramp and smooth ramp programs have been tested on a two parameter family of analytic step responses suggested by Jones and which may be written as

$$F(t) = e^{-\xi t}\left(\cos \Omega t + \frac{(\alpha - 1)\xi}{\Omega} \sin \Omega t\right) \qquad (29)$$

where $\Omega^2 = 1 - \xi^2$; $\alpha \geqslant 1$; $0 < \xi < 1$.

With $\Delta t = 0.5$ and $v = 100$, $\bar{H}$ and $\bar{t}$ were, with the smooth ramp program always obtained to within 5% while $\bar{\gamma}$ was even better (<1%). However, the dominant factor affecting numerical accuracy is the density of tabulated points relative to the scale on which F varies. The period of the above function exceeds 2π by definition of Ω . $\Delta t = 0.5$ is thus not a large spacing by any means. Tests have suggested that the straight ramp program is relatively more accurate probably because of the need to spline fit $\phi(t)$ in the smooth ramp program.

Normally, $F(t)$ would be obtained by solving a set of differential equations in which case Δt would automatically reflect the variability of F so that hopefully no appreciable loss of detail need occur in feeding $F(t)$ to the ramp gust programs. A possible snag is that differential equation solvers tend to vary the step size thereby tabulating $F(t)$ in a manner unacceptable to the smooth ramp program which requires equally spaced points.

In summary, we are confident that <1% error in $\bar{H}$, $\bar{t}$ and $\bar{\gamma}$ is attainable on an ICL 1900 series computer for arbitrary $F(t)$, providing $F(t)$ is well tabulated (*ie* we expect the numerical methods to be able to handle many more than the 200 data points currently allowed by the dimensions of arrays). Obviously, precision will suffer when the programs are run on a machine possessing lower floating point accuracy and double precision working may then become compulsory.

9 USING THE PROGRAMS

There should be little difficulty implementing the programs because they have been written in standard FORTRAN and the data required is minimal. The following quantities need to be input on channel 3.

(a) TMAX the time of the last point in the step response. Set negative if analytic $F(t)$ (programmed in FUNCTION FUNC(T)) is to be used.

(b) TINT the time interval at which $F(t)$ is to be calculated or supplied on channel 1. In the smooth ramp program, TINT is necessarily also the interval at which the ramp response is evaluated. If $F(t)$ is supplied in tabular form to the straight ramp program then TINT is not read and the time points are expected on channel 1 and can be arbitrarily spaced. This facility will probably be little used (but see section 8) because the smooth ramp model is preferred and the same step response data cannot be used immediately.

(c) VELY the speed of the aircraft in ft/s.

(d) DT the time interval at which the straight ramp response is to be calculated. Not read in the smooth ramp program because DT = TINT necessarily.

(e) NH the number of trial values of H supplied ($NH \geqslant 3$)

(f) H(I), I = 1,NH trial gust lengths. $\bar{H}$ to be bracketed; $t_{max} \geqslant H/v \geqslant \Delta t$ preferably (see section 5).

(g) HLTOL accuracy required, $\Delta H/\bar{H}$.

(h) IPR print control parameter giving different levels of output on channel 2 as follows

IPR = - 1 data summary; γ and t_γ for trial $H(I)$ and $\bar{H}$; λ ; λ_w ; worst gust pair:

= 0 as IPR = - 1 plus γ and t_γ for all extra gust lengths investigated including $\frac{1}{2}\bar{H}$ and $2\bar{H}$. Also the integrals in (9) or (10) are output at intervals of Δt for $H = \bar{H}$ and $\frac{1}{2}\bar{H}$.

= 1 as IPR = 0 plus the tabulated spline fit to the step response.

= 2 as IPR = 0 plus ramp responses for each $H(I)$ supplied by the user. The multiplying factor $vH^{-2/3}$ or $\frac{\pi}{2} vH^{-2/3}$ is also output for converting the integrals to $\phi(H,t)$.

= 3 as IPR = 1 and IPR = 2 combined.

(i) IANAL Set to zero if subroutine ANALYTIC to evaluate $\phi(H,t)$ analytically is either not provided or not being used. Otherwise, input any non-zero integer value.

This completes the channel 3 data. If TMAX > 0 then the step response must be tabulated on channel 1. Both programs currently include coding for $F(t)$ given
165 by (29) and the straight ramp version can determine $\phi(H,t)$ by integrating

analytically. They expect α and ξ to be input immediately after TMAX (< 0). Figs 9 to 12 typify the computer output (IPR = 3). The straight ramp program produces similar output except that the columns of the spline fit headed D2F/DT2 and D3F/DT3 are absent because they are not needed.

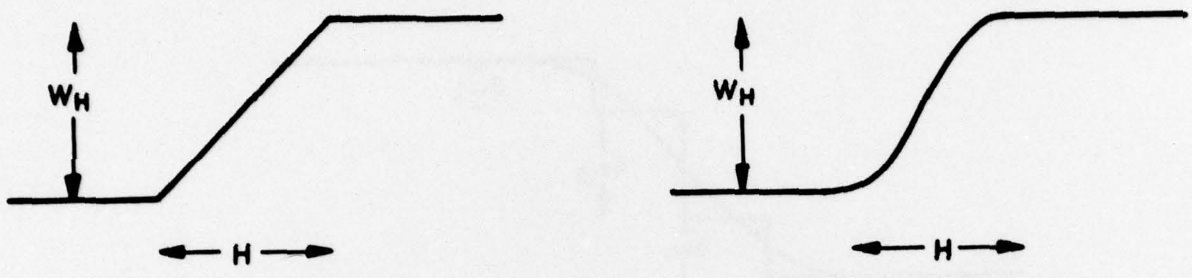
10 CONCLUSIONS

At the request of J.G. Jones of Flight Systems Department, RAE, Bedford, two FORTRAN programs have been written to facilitate the identification of the discrete gusts which produce the greatest aircraft response. The input to the programs is the calculated response as a function of time to a unit step gust. From this one can determine the behaviour on encountering a gust whose intensity profile is of ramp form. The process is in principle quite straightforward but converting the theory into an algorithm suitable for a computer required considerable care. Our declared aim was to reduce to negligible proportions the numerical errors accompanying the implementation so that consistent results can be obtained. This has been achieved at very moderate extra expense in terms of computational effort and thus seems well worthwhile.

The programs are very easy to use and have been applied at RAE, Bedford, to realistic step response profiles with satisfactory results although double precision working was needed because of the shorter wordlength of the SIGMA computer.

REFERENCES

No.	Author	Title, etc
1	J.G. Jones	Statistical discrete gust theory for aircraft loads: a progress report. RAE Technical Report 73167 (1973)
2	J.G. Jones	Modelling of gusts and wind shear for aircraft assessment and certification. Paper presented at the CAARC Symposium of Operational Problems, India (1976)
3	R.P. Brent	*Algorithms for minimisation without derivatives.* Englewood Cliffs, New Jersey, Prentice-Hall Inc (1973)
4	P.E. Gill W. Murray	Safeguarded steplength algorithms for optimisation using descent methods. National Physical Laboratory Report NAC37 (1974)

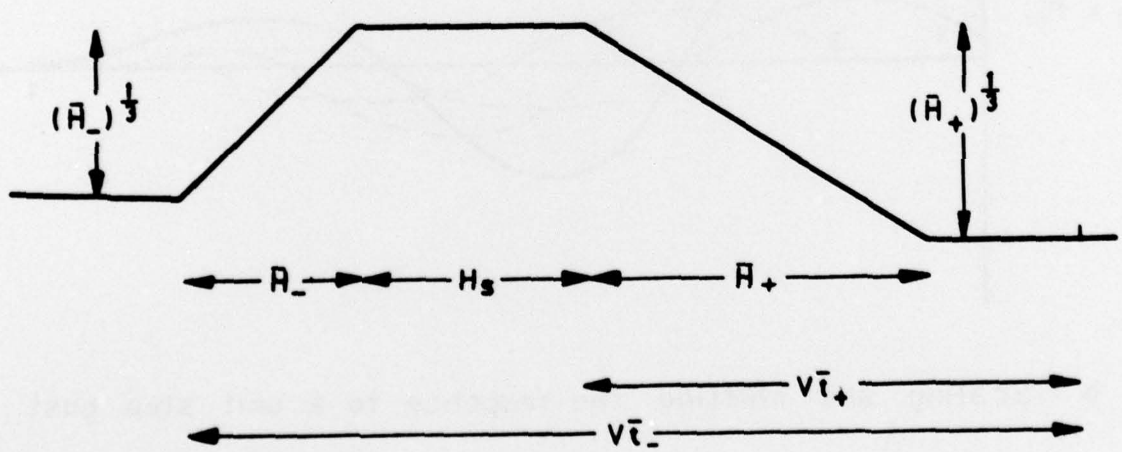


Straight ramp | Smooth ramp

$$W_1(x) = W_H \frac{x}{H}\ ;\ 0 \leqslant x \leqslant H\ ;\ W_2(x) = \frac{1}{2} W_H \left(1 - \cos \frac{\pi x}{H}\right)$$

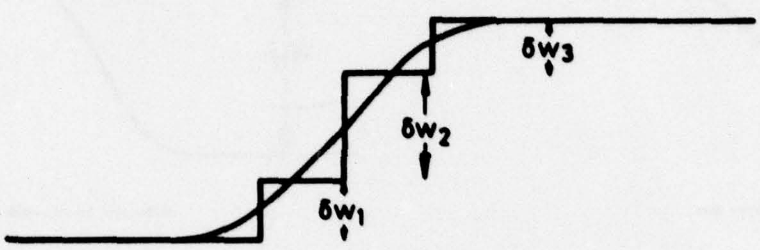
$$W_H = W_0 \ H^{\frac{1}{3}}$$

Fig 1 Model gust intensity profiles

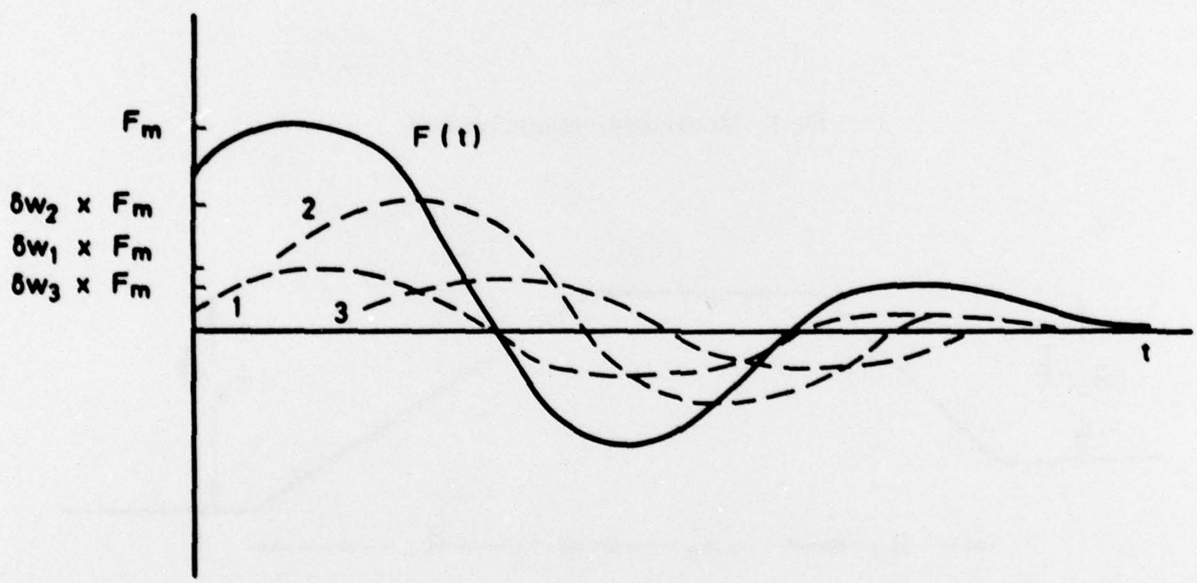


The extreme response will be $(\bar{y}_+ + |\bar{y}_-|)$

Fig 2 Critical gust pair



a Replacing a smooth ramp by a series of steps



b Scaling and shifting the response to a unit step gust

Fig 3a&b Numerical approximation of a (smooth) ramp gust. The curves 1, 2, 3 are then added together to get $\phi(H,t)$. These diagrams are schematic; the actual computation is considerably more sophisticated (see text)

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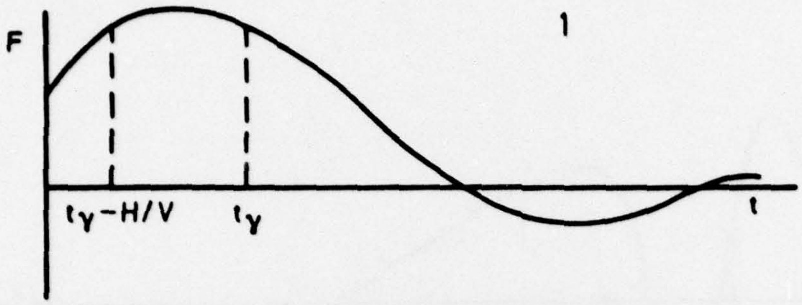
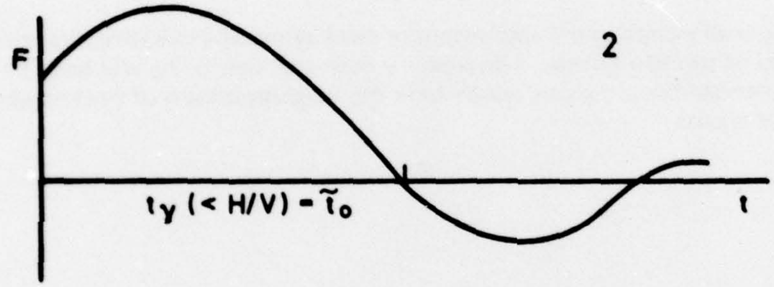
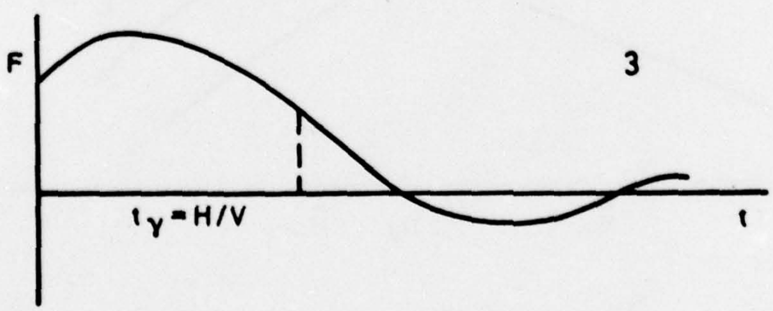


$$\left.\frac{\partial \phi_1}{\partial t}\right|_{t_\gamma} = 0 \text{ because } F(t_\gamma) = F(t_\gamma - H/V)$$



$$\left.\frac{\partial \phi_1}{\partial t}\right|_{t_\gamma} = 0 \text{ because } t_\gamma < H/V \text{ and } F(t_\gamma) = 0$$



$\frac{\partial \phi_1}{\partial t}$ changes sign discontinuously at t_γ because $F(o) > F(t_\gamma) > 0$

Fig 4 The time at which the extreme response to a straight ramp gust occurs (see equation (11) *et seq*)

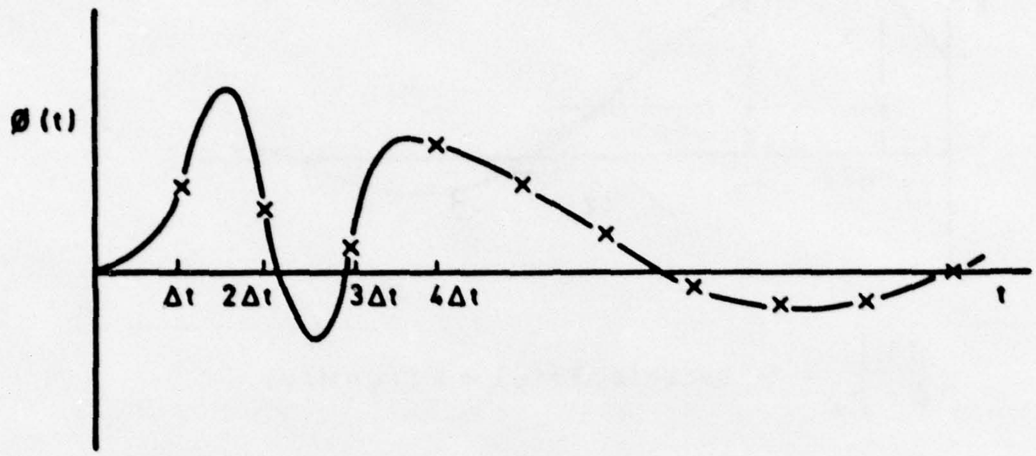


Fig 5 The danger of sampling the step response (and hence of generating the ramp response) at too few points. The primary peak and overswing will both be missed because the programs search only the neighbourhood of the extreme tabulated points

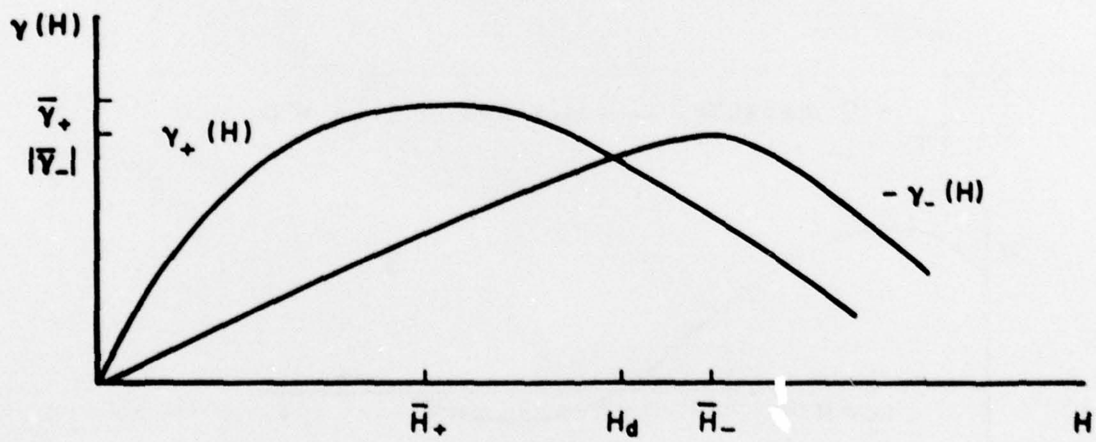
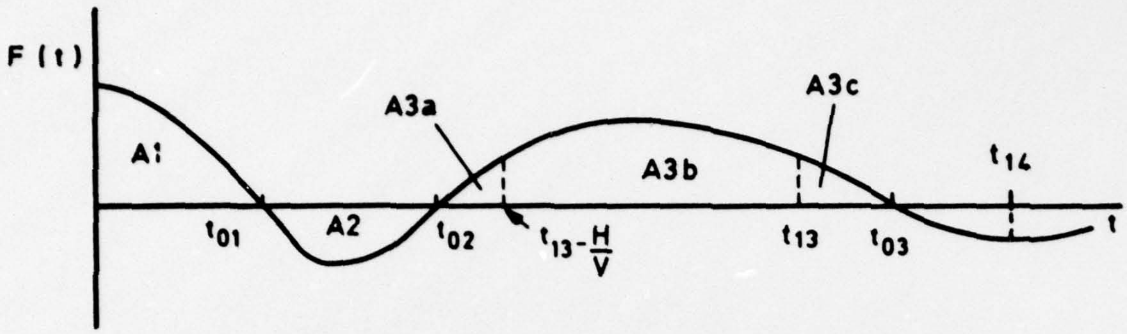
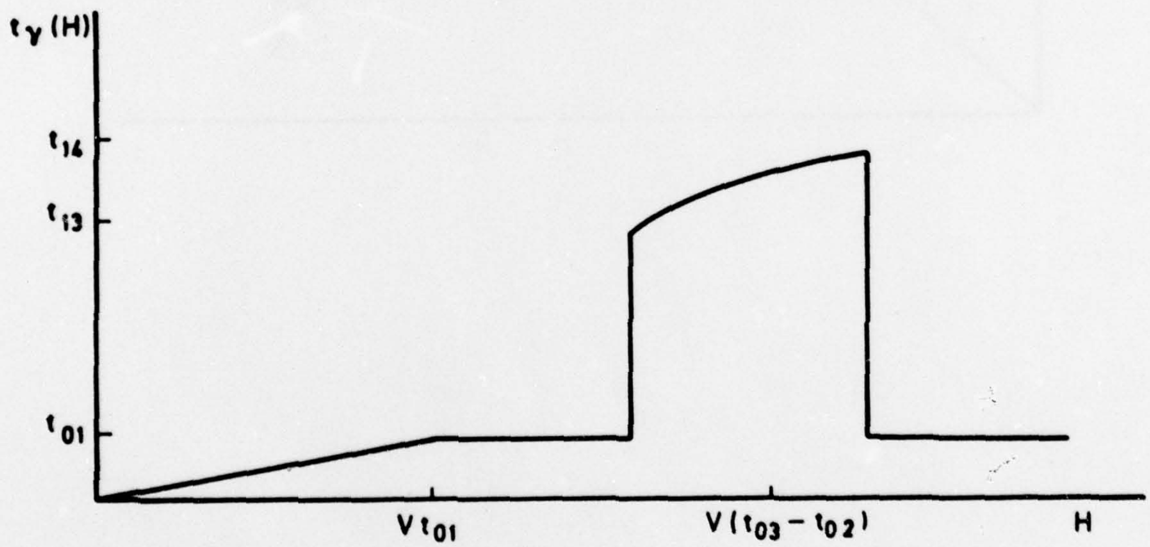


Fig 6 Variation of extreme response with length of gust. Defining γ_+ to be the envelope of the above curves could cause trouble, *eg* the worst gust pair would be wrongly predicted because the peak at $\bar{H}_-$ would be ignored. Alternatively, because γ_+ becomes double humped, $\bar{H}_+$ might be missed



a Step response having the properties
(i) $t_{03}-t_{02}>t_{01}$
(ii) area A3 > area A1 (A3b = A1)



b $t_\gamma(H)$ for the above step response using the straight ramp model

Fig 7a&b Showing possible discontinuous behaviour of $t_\gamma(H)$

Fig 8

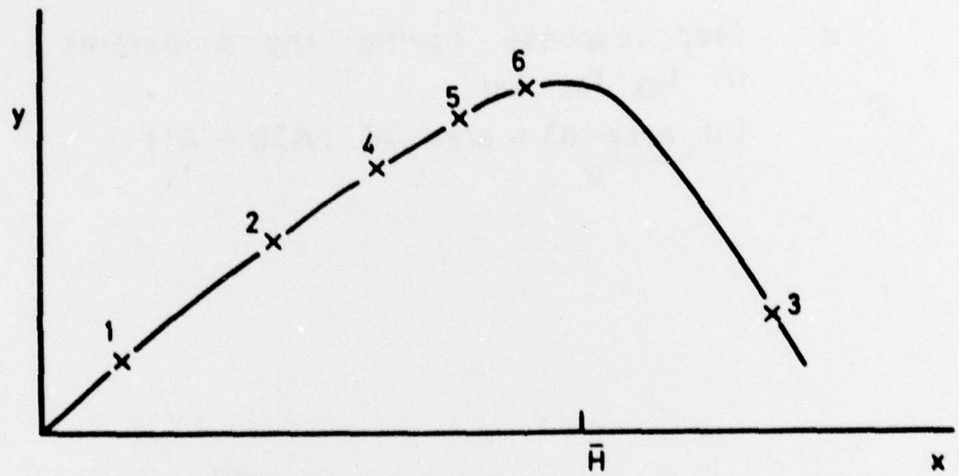


Fig 8 Quadratic interpolation of a skew function. Points 1, 2, 3 are the starting values. Points 4, 5, 6 are obtained by fitting a parabola through (1, 2, 3), (2, 4, 3), (4, 5, 3) respectively always retaining two points bracketing the best (highest). This process is inefficient compared to that described in section 5

```
AIRCRAFT RESPONSE TO SMOOTH RAMP GUSTS      (A.G. PURCELL     APRIL 1976)
--------------------------------------

ANALYTIC STEP RESPONSE DEFINED BY  ALPHA =  5.00  AND  XI = 0.50

AIRCRAFT VELOCITY =  100. FT/SEC

RAMP RESPONSE CALCULATED AT 0.20 SEC INTERVALS UP TO T =  10.00 SECS

GUSTS OF LENGTH H <  20.0 FT ARE EFFECTIVELY STEPS
SUCH H(I) IN THE DATA ARE IGNORED
```

Fig 9 Title page of output from the smooth ramp response program

Fig 10

CUBIC SPLINE FIT TO STEP RESPONSE

T	F(T)	DF/DT	D2F/DT2	D3F/DT3
0.0000E 00	1.0000E 00	1.5015E 00	-2.5304E 00	1.3028E 00
2.0000E-01	1.2514E 00	1.0215E 00	-2.2699E 00	1.3028E 00
4.0000E-01	1.4121E 00	5.9355E-01	-2.0093E 00	1.4763E 00
6.0000E-01	1.4926E 00	2.2122E-01	-1.7140E 00	1.5059E 00
8.0000E-01	1.5045E 00	-9.1469E-02	-1.4129E 00	1.4915E 00
1.0000E 00	1.4600E 00	-3.4421E-01	-1.1146E 00	1.4216E 00
1.2000E 00	1.3707E 00	-5.3869E-01	-8.3023E-01	1.3141E 00
1.4000E 00	1.2481E 00	-6.7845E-01	-5.6741E-01	1.1784E 00
1.6000E 00	1.1027E 00	-7.6837E-01	-3.3173E-01	1.0247E 00
1.8000E 00	9.4372E-01	-8.1422E-01	-1.2679E-01	8.6187E-01
2.0000E 00	7.7949E-01	-8.2234E-01	4.5588E-02	6.9741E-01
2.2000E 00	6.1687E-01	-7.9927E-01	1.8507E-01	5.3755E-01
2.4000E 00	4.6145E-01	-7.5151E-01	2.9258E-01	3.8725E-01
2.6000E 00	3.1750E-01	-6.8525E-01	3.7003E-01	2.5020E-01
2.8000E 00	1.8818E-01	-6.0624E-01	4.2007E-01	1.2896E-01
3.0000E 00	7.5509E-02	-5.1964E-01	4.4586E-01	2.5028E-02
3.2000E 00	-1.9469E-02	-4.2997E-01	4.5087E-01	-6.0965E-02
3.4000E 00	-9.6527E-02	-3.4102E-01	4.3867E-01	-1.2917E-01
3.6000E 00	-1.5613E-01	-2.5586E-01	4.1284E-01	-1.8034E-01
3.8000E 00	-1.9928E-01	-1.7690E-01	3.7677E-01	-2.1572E-01
4.0000E 00	-2.2742E-01	-1.0586E-01	3.3363E-01	-2.3689E-01
4.2000E 00	-2.4223E-01	-4.3874E-02	2.8625E-01	-2.4507E-01
4.4000E 00	-2.4561E-01	8.4623E-03	2.3712E-01	-2.4397E-01
4.6000E 00	-2.3950E-01	5.1006E-02	1.8832E-01	-2.3377E-01
4.8000E 00	-2.2585E-01	8.3995E-02	1.4157E-01	-2.1697E-01
5.0000E 00	-2.0650E-01	1.0797E-01	9.8173E-02	-1.9558E-01
5.2000E 00	-1.8321E-01	1.2370E-01	5.9097E-02	-1.7064E-01
5.4000E 00	-1.5751E-01	1.3210E-01	2.4969E-02	-1.4422E-01
5.6000E 00	-1.3079E-01	1.3421E-01	-3.8745E-03	-1.1738E-01
5.8000E 00	-1.0418E-01	1.3109E-01	-2.7350E-02	-9.1159E-02
6.0000E 00	-7.8628E-02	1.2380E-01	-4.5582E-02	-6.6399E-02
6.2000E 00	-5.4869E-02	1.1335E-01	-5.8861E-02	-4.3729E-02
6.4000E 00	-3.3434E-02	1.0071E-01	-6.7607E-02	-2.3588E-02
6.6000E 00	-1.4676E-02	8.6713E-02	-7.2325E-02	-6.2455E-03
6.8000E 00	1.2115E-03	7.2123E-02	-7.3574E-02	8.1790E-03
7.0000E 00	1.4175E-02	5.7571E-02	-7.1938E-02	1.9693E-02
7.2000E 00	2.4277E-02	4.3578E-02	-6.8000E-02	2.8409E-02
7.4000E 00	3.1671E-02	3.0546E-02	-6.2318E-02	3.4518E-02
7.6000E 00	3.6580E-02	1.8773E-02	-5.5414E-02	3.8272E-02
7.8000E 00	3.9277E-02	8.4554E-03	-4.7760E-02	3.9963E-02
8.0000E 00	4.0066E-02	-2.9733E-04	-3.9767E-02	3.9904E-02
8.2000E 00	3.9264E-02	-7.4527E-03	-3.1786E-02	3.8413E-02
8.4000E 00	3.7189E-02	-1.3042E-02	-2.4104E-02	3.5804E-02
8.6000E 00	3.4147E-02	-1.7146E-02	-1.6943E-02	3.2377E-02
8.8000E 00	3.0422E-02	-1.9887E-02	-1.0468E-02	2.8394E-02
9.0000E 00	2.6273E-02	-2.1413E-02	-4.7888E-03	2.4131E-02
9.2000E 00	2.1927E-02	-2.1888E-02	3.7294E-05	1.9681E-02
9.4000E 00	1.7576E-02	-2.1487E-02	3.9735E-03	1.5653E-02
9.6000E 00	1.3379E-02	-2.0379E-02	7.1040E-03	1.0572E-02
9.8000E 00	9.4591E-03	-1.8747E-02	9.2184E-03	1.0572E-02
1.0000E 01	5.9081E-03	-1.6692E-02	1.1333E-02	1.0572E-02

Fig 10 Spline fit to F(t) . Output only if IPR = 1 or 3

```
RESPONSE TO SMOOTH RAMP GUST OF LENGTH   25.0 FT    (MULTIPLY BY  18.372145 TO GET PHI(T))

 0.0000E 00  1.6058E-01  2.0965E-01  2.3049E-01  2.3906E-01  2.3744E-01  2.2753E-01  2.1140E-01  1.9019E-01  1.6613E-01
 1.4042E-01  1.1430E-01  8.8779E-02  6.4680E-02  4.2612E-02  2.3005E-02  6.1202E-03 -7.9247E-03 -1.9137E-02 -2.7626E-02
-3.3578E-02 -3.7237E-02 -3.8888E-02 -3.8834E-02 -3.7367E-02 -3.4851E-02 -3.1516E-02 -2.7646E-02 -2.3479E-02 -1.9219E-02
-1.5037E-02 -1.1069E-02 -7.4211E-03 -4.1659E-03 -1.3503E-03  1.0037E-03  2.8947E-03  4.3384E-03  5.3656E-03  6.0091E-03
 6.3206E-03  6.3471E-03  6.1396E-03  5.7478E-03  5.2193E-03  4.5980E-03  3.9253E-03  3.2290E-03  2.5440E-03  1.8912E-03
 1.2883E-03


RESPONSE TO SMOOTH RAMP GUST OF LENGTH   50.0 FT    (MULTIPLY BY  11.573726 TO GET PHI(T))

 0.0000E 00  1.2061E-01  3.4798E-01  4.3551E-01  4.6751E-01  4.7696E-01  4.6750E-01  4.4285E-01  4.0666E-01  3.6234E-01
 3.1296E-01  2.6122E-01  2.0943E-01  1.5945E-01  1.1277E-01  7.0480E-02  3.3508E-02  1.6711E-03 -2.4292E-02 -4.4671E-02
-5.9741E-02 -6.9922E-02 -7.5750E-02 -7.7749E-02 -7.6593E-02 -7.2879E-02 -6.7204E-02 -6.0130E-02 -5.2160E-02 -4.3760E-02
-3.5297E-02 -2.7093E-02 -1.9397E-02 -1.2395E-02 -6.2140E-03 -9.2927E-04  3.4311E-03  6.8771E-03  9.4500E-03  1.1215E-02
 1.2255E-02  1.2663E-02  1.2540E-02  1.1986E-02  1.1100E-02  9.9728E-03  8.6898E-03  7.3254E-03  5.9430E-03  4.5975E-03
 3.3293E-03


RESPONSE TO SMOOTH RAMP GUST OF LENGTH  100.0 FT    (MULTIPLY BY   7.290991 TO GET PHI(T))

 0.0000E 00  6.6446E-02  2.5873E-01  5.2304E-01  7.6833E-01  9.0248E-01  9.3541E-01  9.2858E-01  8.8942E-01  8.2523E-01
 7.4286E-01  6.4858E-01  5.4795E-01  4.4576E-01  3.4599E-01  2.5180E-01  1.6559E-01  8.9064E-02  2.3119E-02 -3.1675E-02
-7.5392E-02 -1.0846E-01 -1.3161E-01 -1.4582E-01 -1.5218E-01 -1.5168E-01 -1.4616E-01 -1.3619E-01 -1.2310E-01 -1.0794E-01
-9.1629E-02 -7.4963E-02 -5.8609E-02 -4.3102E-02 -2.8848E-02 -1.6136E-02 -5.1449E-03  4.0396E-03  1.1413E-02  1.7038E-02
 2.1027E-02  2.3534E-02  2.4730E-02  2.4826E-02  2.4003E-02  2.2461E-02  2.0388E-02  1.7934E-02  1.5312E-02  1.2596E-02
 9.9166E-03


RESPONSE TO SMOOTH RAMP GUST OF LENGTH  200.0 FT    (MULTIPLY BY   4.593036 TO GET PHI(T))

 0.0000E 00  3.4029E-02  1.4224E-01  3.2410E-01  5.6609E-01  8.4765E-01  1.1362E 00  1.3987E 00  1.6021E 00  1.7172E 00
 1.7230E 00  1.6428E 00  1.5177E 00  1.3605E 00  1.1826E 00  9.9428E-01  8.0413E-01  6.1939E-01  4.4576E-01  2.8750E-01
 1.4754E-01  2.7616E-02 -7.1567E-02 -1.5018E-01 -2.0912E-01 -2.4982E-01 -2.7412E-01 -2.8611E-01 -2.8202E-01 -2.7012E-01
-2.5061E-01 -2.2559E-01 -1.9695E-01 -1.6636E-01 -1.3535E-01 -1.0505E-01 -7.6445E-02 -5.0264E-02 -2.7013E-02 -7.0001E-03
 9.6378E-03  2.2911E-02  3.2931E-02  3.9980E-02  4.4291E-02  4.6220E-02  4.6129E-02  4.4387E-02  4.1338E-02  3.7383E-02
 3.2778E-02


RESPONSE TO SMOOTH RAMP GUST OF LENGTH  400.0 FT    (MULTIPLY BY   2.893432 TO GET PHI(T))

 0.0000E 00  1.7116E-02  7.2808E-02  1.7077E-01  3.1113E-01  4.9086E-01  7.0416E-01  9.4300E-01  1.1977E 00  1.4574E 00
 1.7109E 00  1.9466E 00  2.1538E 00  2.3224E 00  2.4438E 00  2.5111E 00  2.5168E 00  2.4640E 00  2.3450E 00  2.1645E 00
 1.9240E 00  1.6461E 00  1.3590E 00  1.0749E 00  8.0351E-01  5.5224E-01  3.2656E-01  1.3000E-01 -3.5630E-02 -1.6996E-01
-2.7379E-01 -3.4891E-01 -3.9780E-01 -4.2346E-01 -4.2916E-01 -4.1833E-01 -3.9434E-01 -3.6043E-01 -3.1968E-01 -2.7475E-01
-2.2804E-01 -1.8155E-01 -1.3693E-01 -9.5455E-02 -5.8046E-02 -2.5320E-02  2.3883E-03  2.4988E-02  4.2587E-02  5.5458E-02
 6.3992E-02
```

Fig 11 Values of the integral in equation (10) at intervals of $\Delta t = 0.2$ s.
Output if IPR = 2 or 3

ESTIMATED EXTREMES OF RESPONSE TO RAMP GUSTS

INDIC	GUST LENGTH	GAMMA(+)	T(+)	GAMMA(-)	T(-)
0	25.00	4.4011E 00	8.6373E-01	-7.1754E-01	4.4914E 00
0	50.00	5.5207E 00	9.9151E-01	-9.0004E-01	4.6409E 00
0	100.00	6.8271E 00	1.2555E 00	-1.1140E 00	4.8887E 00
0	200.00	7.9611E 00	1.9088E 00	-1.3074E 00	5.4609E 00
0	400.00	7.3005E 00	3.1257E 00	-1.2426E 00	6.7628E 00
0	219.15	8.0111E 00	2.0425E 00		
0	233.61	8.0245E 00	2.1483E 00		
0	237.47	8.0227E 00	2.1729E 00		
0	231.31	8.0243E 00	2.1355E 00		
0	235.52	8.0239E 00	2.1604E 00		

Gust lengths, H(I) specified by the user

Interpolated estimates of $\bar{H}_+$. Output unless IPR = −1

CONVERGENCE CRITERION SATISFIED : DH/HBAR < 0.0099
REQUIRED ACCURACY = 0.010

IT= 5	233.61	8.0245E 00	2.1483E 00
	******	**********	**********

$\bar{H}_+, \bar{\gamma}_+, \bar{t}_+$

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 233.6 FT (MULTIPLY BY 4.141138 TO GET PHI(T))

```
 0.0000E 00  2.9195E-02  1.2279E-01  2.8271E-01  5.0179E-01  7.6498E-01  1.0510E 00  1.3346E 00  1.5889E 00  1.7879E 00
 1.9087E 00  1.9340E 00  1.8560E 00  1.7157E 00  1.5388E 00  1.3384E 00  1.1259E 00  9.1132E-01  7.0267E-01  5.0645E-01
 3.2751E-01  1.6916E-01  3.3409E-02 -7.8944E-02 -1.6808E-01 -2.3498E-01 -2.8127E-01 -3.0900E-01 -3.2036E-01 -3.1841E-01
-3.0515E-01 -2.8328E-01 -2.5513E-01 -2.2287E-01 -1.8840E-01 -1.5337E-01 -1.1915E-01 -8.6851E-02 -5.7231E-02 -3.0928E-02
-8.2752E-03  1.0570E-02  2.5617E-02  3.7011E-02  4.5003E-02  4.9919E-02  5.2141E-02  5.2076E-02  5.0141E-02  4.6745E-02
 4.2276E-02
```

½$\bar{H}$

RESPONSE TO SMOOTH RAMP GUST OF LENGTH 116.8 FT (MULTIPLY BY 6.573647 TO GET PHI(T))

```
 0.0000E 00  5.7373E-02  2.2915E-01  4.8378E-01  7.5790E-01  9.7545E-01  1.0718E 00  1.0842E 00  1.0551E 00  9.9320E-01
 9.0663E-01  8.0298E-01  6.8911E-01  5.7098E-01  4.5304E-01  3.4117E-01  2.3676E-01  1.4272E-01  6.0577E-02 -8.8094E-03
-6.5406E-02 -1.0933E-01 -1.4134E-01 -1.6245E-01 -1.7385E-01 -1.7691E-01 -1.7302E-01 -1.6359E-01 -1.4995E-01 -1.3337E-01
-1.1497E-01 -9.5757E-02 -7.6564E-02 -5.8084E-02 -4.0856E-02 -2.5274E-02 -1.1603E-02  8.9802E-06  9.5156E-03  1.6955E-02
 2.2434E-02  2.6108E-02  2.8173E-02  2.8846E-02  2.8355E-02  2.6928E-02  2.4787E-02  2.2138E-02  1.9170E-02  1.6046E-02
 1.2910E-02
```

0	116.81	7.1397E 00	1.3398E 00
0	467.23	6.8627E 00	3.4562E 00

LAMBDA (W) = 0.270 LAMBDA = 0.291 from equations (26) and (25)

Fig 12 Results page

```
   0     237.69                                   -1.3319E 00    5.6896E 00
   0     255.06                                   -1.3363E 00    5.7975E 00
   0     270.85                                   -1.3368E 00    5.8970E 00
   0     265.91                                   -1.3370E 00    5.8657E 00
   0     263.29                                   -1.3369E 00    5.8492E 00
   0     268.35                                   -1.3369E 00    5.8811E 00
```

Interpolated estimates of $\bar{H}_-$.
Output unless IPR = −1.

```
CONVERGENCE CRITERION SATISFIED :    DH/HBAR <  0.0099
                        REQUIRED ACCURACY =  0.010

IT= 6    265.91                                   -1.3370E 00    5.8657E 00
         ******                                   **********     **********
```

$\bar{H}_-, \bar{\gamma}_-, \bar{t}_-$

```
RESPONSE TO SMOOTH RAMP GUST OF LENGTH  265.9 FT     (MULTIPLY BY    3.798638 TO GET PHI(T))

 0.0000E 00  2.5683E-02  1.0845E-01  2.5129E-01  4.5010E-01  6.9445E-01  9.6871E-01  1.2535E 00  1.5272E 00  1.7679E 00
 1 9547E 00  2.0695E 00  2.0984E 00  2.0344E 00  1.8809E 00  1.6875E 00  1.4678E 00  1.2351E 00  9.9994E-01  7.7126E-01
 5 5616E-01  3.5996E-01  1.8632E-01  3.7425E-02 -8.5834E-02 -1.8365E-01 -2.5710E-01 -3.0794E-01 -3.3844E-01 -3.5119E-01
-3.4893E-01 -3.3447E-01 -3.1035E-01 -2.7975E-01 -2.4442E-01 -2.0666E-01 -1.6828E-01 -1.3078E-01 -9.5346E-02 -6.2892E-02
-3 4049E-02 -9.2041E-03  1.1470E-02  2.7982E-02  4.0490E-02  4.9267E-02  5.4674E-02  5.7124E-02  5.7067E-02  5.4957E-02
 5.1245E-02
```

½$\bar{H}_-$

```
RESPONSE TO SMOOTH RAMP GUST OF LENGTH  133.0 FT     (MULTIPLY BY    6.029962 TO GET PHI(T))

 0 0000E 00  5.0675E-02  2.0565E-01  4.4587E-01  7.2631E-01  9.8667E-01  1.1658E 00  1.2224E 00  1.2095E 00  1.1553E 00
 1 0692E 00  9.5999E-01  8.5592E-01  7.0414E-01  5.7081E-01  4.4102E-01  3.1882E-01  2.0725E-01  1.0842E-01  2.3383E-02
-4 6727E-02 -1.0260E-01 -1.4404E-01 -1.7384E-01 -1.9146E-01 -1.9896E-01 -1.9792E-01 -1.8992E-01 -1.7650E-01 -1.5914E-01
-1 3917E-01 -1.1750E-01 -9.0045E-02 -7.4700E-02 -5.4634E-02 -3.6182E-02 -1.9767E-02 -5.6133E-03  6.1775E-03  1.5608E-02
 2 2765E-02  2.7802E-02  3.0921E-02  3.2338E-02  3.2365E-02  3.1201E-02  2.9121E-02  2.6366E-02  2.3137E-02  1.9693E-02
 1.6146E-02



   0     132.96                                   -1.2029E 00    5.0718E 00
   0     531.82                                   -1.0333E 00    7.7508E 00

   LAMBDA (W) =  0.258        LAMBDA =  0.329



   ESTIMATED WORST RESPONSE TO AN ISOLATED GUST,    8.0245E 00
                                     OCCURS WHEN H =  233.6

   ESTIMATED WORST RESPONSE TO A PAIR OF GUSTS,     9.3615E 00

   WHICH OCCURS WHEN THE FIRST GUST IS OF LENGTH, H1 =  265.9

           AND THE SECOND GUST IS OF LENGTH, H2 =  233.6

       THE TWO BEING SEPARATED BY A DISTANCE, HS =  105.8
```

Max $(\bar{\gamma}_+, -\bar{\gamma}_-)$

$\bar{\gamma}_+ - \bar{\gamma}_-$

$\bar{H}_-$ (because $\bar{t}_- > \bar{t}_+$)

$\bar{H}_+$

from equation (28)

Fig 12 (contd) Results page